Viscoelastic modeling of articular cartilage under impact loading

Roberta Springhetti[1], N.S. Selyutina[2]
[1]*Department of Civil, Environmental and Mechanical Engineering*
E-mail: roberta.springhetti@unitn.it

[2]*St. Petersburg State University, 198504 St. Petersburg, Russia*
E-mail: nina.selutina@gmail.com

Keywords: Quasi–linear viscoelasticity, impact, soft tissues.

The occurrence of impulsive mechanical loads on articular cartilage can result in a damage of the tissue, which can in turn be at the origin of degenerative processes as osteoarthritis. This evidence has motivated a number of experimental studies on articular cartilage undergoing high rate loading: with reference to the recent experimental investigation carried out by Burgin et al. [1] under drop impact loading, the present contribution aims to develop an analytical model to describe the unconfined impact test on specimens of articular cartilage samples, shaped as cylinders. The authors of the experimental tests in [1] had already provided a mathematical approach, followed by a comprehensive study carried out by Selyutina et al. [3] on the basis of small deformation theory: as the authors highlighted, in order to better explain the experimental behaviour of cartilage, involving very large strain values, finite viscoelasticity is needed, what actually has been adopted here.

We assume that a cartilage cylindrical sample, having initial thickness H and cross-sectional area $A = \pi R^2$, is supported by a rigid surface along the plane $X_3 = 0$ (according to a Lagrangian coordinate system, whose origin is at the centre of the lower face of the sample) and interacts without friction along the plane $X_3 = H$ with a rigid impactor, having mass m^I and freely falling along the negative direction of axis X_3. If $x^I(t)$ denotes the current distance between the lower surface of the impactor and the plane supporting the cartilage and $H^I = H + H_0$ represents its initial value, the collision takes place at time t_0 with $x^I(t_0) = H$ and initial velocity $\dot{x}^I(t_0) = -\sqrt{2gH_0}$, being g the gravitational acceleration. As we are interested in the description of the impact itself, we assume $t_0 = 0$ and study the motion of the impactor ($t \geq 0$), deforming the cartilage and remaining in contact with it, until it is eventually bounced back at time $t = t_c$.

Articular cartilage is actually a multiphasic material with a complex structure and a significant viscoelastic behaviour; a large variety of mechanical models have been proposed to explain its experimental stress-strain behaviour, essentially based on a nonlinear biphasic poroviscoelastic model. Since the impact test is very rapid, the contained water has less time to flow outside the sample, as confirmed by the absence of any measurable loss of mass during the tests of Burgin et al. [1], therefore cartilage is modelled here as an incompressible homogeneous medium. The deformation gradient is in principle $\mathbf{F}(t) = diag\{\lambda_r(t), \lambda_r(t), \lambda(t)\}$, with stretches $\lambda(t)$ and $\lambda_r(t)$ defining current thickness and area of the sample, $h(t) = \lambda(t)H$ and $a(t) = \lambda_r^2(t)A$, respectively. Under the hypothesis of incompressibility, $J(t) = \det \mathbf{F}(t) = 1$, therefore $\lambda_r(t) = \lambda^{-1/2}(t)$.

In order to model the visco-elastic behaviour of articular cartilage, the Fung's model, recently reappraised and reformulated by De Pascalis et al. [2], has been assumed, adopting the stress-relaxation function $G(t) = 1 + \delta(t/\tau_R)$, corresponding to the Kelvin-Voigt rhelogical model, as shown in [3]. The current Cauchy stress $\boldsymbol{\sigma}(t)$ can thus be expressed through its instantaneous component $\boldsymbol{\sigma}^e[\mathbf{b}(t)]$, being a function of the left Cauchy-Green tensor $\mathbf{b} = \mathbf{F}\mathbf{F}^T$, as

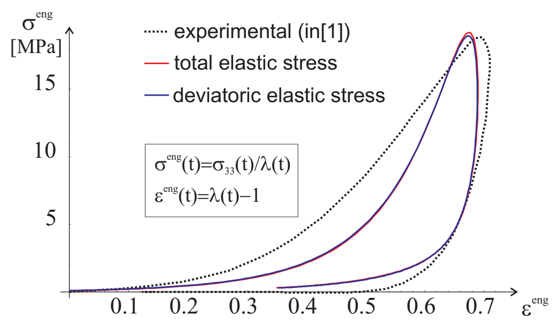

Figure 1: Comparison between experimental and analytical results for Raghavan–Vorp model with $\alpha = 1.6$, $\mu_0 = 0.22\,\mathrm{MPa}$, $\tau_R = 0.175\,\mathrm{ms}$ and $\tau_R = 0.137\,\mathrm{ms}$ for blue and red curve, respectively.

$$\boldsymbol{\sigma}(t) = -p(t)\mathbf{I} + \hat{\boldsymbol{\sigma}}^e[\mathbf{b}(t)] + \mathbf{F}(t)\left[\int_0^t \frac{\partial \mathcal{G}(t-\tau)}{\partial(t-\tau)} \mathbf{F}^{-1}(\tau)\, \hat{\boldsymbol{\sigma}}^e[\mathbf{b}(\tau)]\mathbf{F}^{-T}(\tau)\, d\tau\right]\mathbf{F}^T(t). \qquad (1)$$

where $\hat{\boldsymbol{\sigma}}^e[\mathbf{b}(t)]$ is assumed to be either $\boldsymbol{\sigma}^e[\mathbf{b}(t)]$ or its deviatoric component, as suggested in [2] for incompressible materials, while $p(t)$ represents the hydrostatic pressure, evaluated through boundary conditions $\sigma_{11}(t) = \sigma_{22}(t) = 0$. Assuming that cartilage behaves as a hyperelastic isotropic material, the instantaneous stress is derived from a strain energy function $W(t) = W(I_1[\mathbf{b}(t)], I_2[\mathbf{b}(t)])$, only depending on the invariants $I_1[\mathbf{b}] = \operatorname{tr}\mathbf{b}$ and $I_2[\mathbf{b}] = 1/2[(\operatorname{tr}\mathbf{b})^2 - \operatorname{tr}\mathbf{b}^2]$, while for incompressibility, $I_3[\mathbf{b}] = \det\mathbf{b} = 1$. The Raghavan-Vorp model, a 2 parameters Yeoh-type model, has been chosen, which is based on the strain energy function $W(I_1) = \frac{\mu_0}{4}\left[2(I_1 - 3) + \alpha(I_1 - 3)^2\right]$. Fig. 1 shows a comparison between the experimental curve obtained by Burgin et al. in [1] for $m^I = 100\,\mathrm{g}$, $H = 1.42\,\mathrm{mm}$, $R = 2.5\,\mathrm{mm}$ and $H_0 = 80\,\mathrm{mm}$ and the analytical results corresponding to a suitable identification of the material parameters μ_0, α and the relaxation time τ_R. Despite the simplicity of the constitutive model, the correspondence is promising and improves the approximation provided in [3]; the approaches based on deviatoric or total instantaneous stresses give close results with an appropriate choice of relaxation time τ_R. A good estimate is also provided for the coefficient of restitution, around 0.506, to be compared with an experimental mean value of 0.502.

Acknowledgements R.S. gratefully acknowledges financial supports from ERC-2013-ADG-340561-INSTABILITIES. N.S. gratefully acknowledges financial support from FP7 IRSES Marie Curie grant TAMER (PIRSES-GA-2013-610547).

References

[1] Burgin, L.V., Edelsten, L. , Aspden R.M., "The mechanical and material properties of elderly human articular cartilage subject to impact and slow loading", Medical Engineering and Physics, 36, 226–232 (2014).

[2] De Pascalis, R., Abrahams, I.D., Parnell, W.J., "On nonlinear viscoelastic deformations: a reappraisal of Fung's quasi-linear viscoelastic model", Proc. R. Soc. A 470: 20140058 (2014).

[3] Selyutina, N.S., Argatov, I.I., Mishuris, G.S., "On application of Fung's quasi-linear viscoelastic model to modeling of impact experiment for articular cartilage", Mech. Res. Comm. 67, 2430 (2015).